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D-deformed Wess-Zumino model and its renormalizability properties

Marija Dimitrijević and Voja Radovanović

University of Belgrade, Faculty of Physics,
Studentski trg 12, 11000 Belgrade, Serbia

E-mail: dmarija@phy.bg.ac.yu, rvoja@phy.bg.ac.yu

Abstract: Using the methods developed in earlier papers we analyze a new type of deformation of the superspace. The twist we use to deform the $N = 1$ SUSY Hopf algebra is non-hermitian and is given in terms of the covariant derivatives D_α. A SUSY invariant deformation of the Wess-Zumino action is constructed and compared with results already known in the literature. Finally, by calculating divergences of the two-point Green functions a preliminary analysis of renormalizability properties of the constructed model is done. As expected, there is no renormalization of mass and no tadpole diagrams appear.

Keywords: Superspaces, Non-Commutative Geometry, Renormalization Regularization and Renormalons

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Contents

1 Introduction

The idea of noncommuting spacetime coordinates goes back to Heisenberg who suggested [1] that uncertainty relations between coordinates could resolve the ultraviolet (UV) divergences arising in quantum field theories. The issue was then investigated by Snyder in [2], but did not attract much interest at the time. However, in the last two decades noncommutative geometry has found applications in many branches of physics such as quantum field theory and particle physics, solid state physics and many others. Comprehensive reviews on the subject can be found in references [3–9].

Having in mind problems which physics encounters at small scales (high energies), in recent years attempts were made to combine supersymmetry (SUSY) with noncommutative geometry. Different models were constructed, see for example [10–15]. Some of these models emerge naturally as low energy limits of string theories in backgrounds with a constant Neveu-Schwarz two form and/or a constant Ramond-Ramond two form. In [13] the anticommutation relations between the fermionic coordinates were modified in the following way

$$\{\theta^\alpha \overset{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \overset{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \overset{\star}{,} \bar\theta_{\dot\alpha}\} = 0\,, \tag{1.1}$$

where $C^{\alpha\beta} = C^{\beta\alpha}$ is a complex constant symmetric matrix. The analysis is done in Euclidean space, since the deformation (1.1) is hermitian only in Euclidean signature where undotted and dotted spinors are not related by the usual complex conjugation. Note that the $\star$-product used in (1.1) is also well defined in Minkowskian signature although in that case it is not hermitian [14]. The chiral coordinates $y^m = x^m + i\theta\sigma^m\bar{\theta}$ commute in this setting, therefore the notion of chirality is preserved, i.e. the $\star$-product of two chiral superfields is again a chiral superfield. On the other hand, the constructed models break one half of the $N = 1$ SUSY so they are invariant only under the so-called $N = 1/2$ SUSY. Renormalizability of the Wess-Zumino models with this deformation was considered, see for example [16]. Some of the obtained results are: the renormalizability is lost already at the one loop level, but it can be restored by adding to the classical action new couplings (interaction terms) which depend on the deformation parameter. Also, a pure gauge sector which is supergauge invariant and one loop renormalizable can be constructed [17]. Recently, a renormalizable $N = 1/2$ super-Yang-Mills model with interacting matter was constructed in [18].

Another type of deformation was introduced in [14]. There the product of two chiral superfields is not a chiral superfield but the model is invariant under the full supersymmetry. A deformation of the Hopf algebra of SUSY transformations by a twist was considered in [19].

In our previous paper [20] we applied the twist formalism to deform the Hopf algebra of $N = 1$ SUSY transformations. Our choice of the twist is different from that in [19]. We work in Minkowski space-time and choose a hermitian twist. As undotted and dotted spinors are related by the usual complex conjugation, we obtain

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar{\theta}_{\dot{\alpha}} \stackrel{\star}{,} \bar{\theta}_{\dot{\beta}}\} = \bar{C}_{\dot{\alpha}\dot{\beta}}, \quad \{\theta^\alpha \stackrel{\star}{,} \bar{\theta}_{\dot{\alpha}}\} = 0\,, \tag{1.2}$$

with $\bar{C}_{\dot{\alpha}\dot{\beta}} = (C_{\alpha\beta})^*$. The deformed Wess-Zumino Lagrangian was formulated and analyzed; the action which follows is invariant under the twisted SUSY transformations. Superfields transform in the undeformed way, while the Leibniz rule for SUSY transformations (when they act on a product of superfields) is modified. Since the action is non-local it is difficult (but not impossible) to discuss renormalizability properties of the model.

Nevertheless, we are interested in renormalizability properties of theories with twisted symmetries. It is important to understand whether deforming (by a twist) of symmetries spoils some of the renormalizability properties of SUSY invariant theories. Therefore, in this paper we analyze a simpler model with the twist given by

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}, \tag{1.3}$$

where $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ is a complex constant matrix and $D_\alpha = \partial_\alpha - i\sigma_m^{\alpha\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}\partial_m$ are the SUSY covariant derivatives.

Following the method of [20] in the next section we introduce the deformation and the $\star$-product which follows from it. Due to our choice of the twist (1.3), the coproduct of SUSY transformations[1] remains undeformed, leading to the undeformed Leibniz rule. Being interested in deformations of the Wess-Zumino model, we discuss chiral fields and their products. The product of two chiral fields is not a chiral field and we have to use the projectors defined in [21] to separate chiral and antichiral parts. All possible invariants are

[1]We only consider the $N = 1$ SUSY in this paper. The generalization to $N = 2$ SUSY and higher can be obtained by following the same steps as in section 2.

listed in section 4 and the deformed Wess-Zumino action is constructed in section 5. Using the background field method we then analyze two-point functions and their divergences. Finally, we give some comments and compare our results with the results already present in the literature. Some details of the calculations are collected in appendices A and B.

2 D-deformation of the Hopf algebra of SUSY transformations

The undeformed superspace is generated by the coordinates x, θ and $\bar\theta$ which fulfill

$$\begin{aligned} [x^m, x^n] &= [x^m, \theta^\alpha] = [x^m, \bar\theta_{\dot\alpha}] = 0, \\ \{\theta^\alpha, \theta^\beta\} &= \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = \{\theta^\alpha, \bar\theta_{\dot\alpha}\} = 0, \end{aligned} \tag{2.1}$$

with $m = 0, \dots 3$ and $\alpha, \beta = 1, 2$. To x^m we refer as to bosonic and to θ^α and $\bar\theta_{\dot\alpha}$ we refer as to fermionic coordinates. Also, $x^2 = x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$. A general superfield $F(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$

$$\begin{aligned} F(x,\theta,\bar\theta) &= f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \theta\sigma^m\bar\theta v_m \\ &\quad + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \tag{2.2}$$

Under the infinitesimal SUSY transformations it transforms in the following way

$$\delta_\xi F = \big(\xi Q + \bar\xi\bar Q\big)F, \tag{2.3}$$

where ξ and $\bar\xi$ are constant anticommuting parameters and Q and $\bar Q$ are the SUSY generators

$$Q_\alpha = \partial_\alpha - i\sigma^m_{\ \alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m, \tag{2.4}$$

$$\bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} - i\theta^\alpha\sigma^m_{\ \alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \tag{2.5}$$

As in [20, 22], we introduce a deformation of the Hopf algebra of infinitesimal SUSY transformations by choosing the twist $\mathcal{F}$ in the following way

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{2.6}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$. Note that this twist is not hermitian, $\mathcal{F}^* \neq \mathcal{F}$. The usual complex conjugation is denoted by "$*$". It can be shown [23] that (2.6) satisfies all the requirements for a twist [24]. The Hopf algebra of SUSY transformation does not change since

$$\{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, D_\beta\} = 0 \tag{2.7}$$

and it is given by

- algebra

$$\begin{aligned} &\{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0, \qquad \{Q_\alpha, \bar Q_{\dot\beta}\} = 2i\sigma^m_{\ \alpha\dot\beta}\partial_m, \\ &[\partial_m, \partial_n] = [\partial_m, Q_\alpha] = [\partial_m, \bar Q_{\dot\alpha}] = 0. \end{aligned} \tag{2.8}$$

- coproduct

$$\Delta Q_\alpha = Q_\alpha \otimes 1 + 1 \otimes Q_\alpha, \qquad \Delta \bar{Q}_{\dot\alpha} = \bar{Q}_{\dot\alpha} \otimes 1 + 1 \otimes \bar{Q}_{\dot\alpha},$$
$$\Delta \partial_m = \partial_m \otimes 1 + 1 \otimes \partial_m. \tag{2.9}$$

- counit and antipode

$$\varepsilon(Q_\alpha) = \varepsilon(\bar{Q}_{\dot\alpha}) = \varepsilon(\partial_m) = 0,$$
$$S(Q_\alpha) = -Q_\alpha, \qquad S(\bar{Q}_{\dot\alpha}) = -\bar{Q}_{\dot\alpha}, \qquad S(\partial_m) = -\partial_m. \tag{2.10}$$

This means that the full supersymmetry is preserved.

Strictly speaking, the twist (2.6) does not belong to the universal enveloping algebra of the Lie algebra of infinitesimal SUSY transformations. Therefore, to be mathematically correct we should enlarge the algebra (2.8) by introducing the relations for the operators D_α as well. Note that the same happened in [20], where the twist was given by

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha \otimes \partial_\beta + \frac{1}{2}\bar{C}_{\dot\alpha\dot\beta}\bar{\partial}^{\dot\alpha}\otimes\bar{\partial}^{\dot\beta}}, \tag{2.11}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha}$ and $C^{\alpha\beta}$ and $\bar{C}^{\dot\alpha\dot\beta}$ were related by the usual complex conjugation. There we had to enlarge the algebra by adding the relations for the fermionic derivatives ∂_α and $\bar{\partial}^{\dot\alpha}$.

The inverse of the twist (2.6)

$$\mathcal{F}^{-1} = e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{2.12}$$

defines the $\star$-product. For two arbitrary superfields F and G the $\star$-product reads

$$\begin{aligned} F \star G &= \mu_\star\{F \otimes G\} \\ &= \mu\{\mathcal{F}^{-1}\, F \otimes G\} \\ &= \mu\{e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta} F \otimes G\} \\ &= F \cdot G - \frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(D_\alpha F)\cdot(D_\beta G) \\ &\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(D_\alpha D_\gamma F)\cdot(D_\beta D_\delta G), \end{aligned} \tag{2.13}$$

where $|F| = 1$ if F is odd (fermionic) and $|F| = 0$ if F is even (bosonic). The second line of (2.13) is the definition of the $\mu_\star$ multiplication. No higher powers of $C^{\alpha\beta}$ appear since the derivatives D_α are Grassmanian. The $\star$-product (2.13) is associative,[2] noncommutative and in the zeroth order in the deformation parameter $C_{\alpha\beta}$ it reduces to the usual pointwise multiplication. One should also note that it is not hermitian,

$$(F \star G)^* \neq G^* \star F^*. \tag{2.15}$$

[2] The associativity of the $\star$-product follows from the cocycle condition [24] which the twist $\mathcal{F}$ has to fulfill

$$\mathcal{F}_{12}(\Delta \otimes id)\mathcal{F} = \mathcal{F}_{23}(id \otimes \Delta)\mathcal{F}, \tag{2.14}$$

where $\mathcal{F}_{12} = \mathcal{F} \otimes 1$ and $\mathcal{F}_{23} = 1 \otimes \mathcal{F}$. It can be shown that the twist (2.6) indeed fulfills this condition, see for details [23].

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The $\star$-product (2.13) leads to

$$\begin{aligned}
&\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0,\\
&[x^m \stackrel{\star}{,} x^n] = -C^{\alpha\beta}(\sigma^{mn}\varepsilon)_{\alpha\beta}\bar\theta\bar\theta,\\
&[x^m \stackrel{\star}{,} \theta^\alpha] = -iC^{\alpha\beta}\sigma^m_{\beta\dot\beta}\bar\theta^{\dot\beta}, \quad [x^m \stackrel{\star}{,} \bar\theta_{\dot\alpha}] = 0.
\end{aligned} \tag{2.16}$$

The chiral coordinates y^m also do not commute

$$[y^m \stackrel{\star}{,} y^n] = -8\bar\theta\bar\theta C^{\alpha\beta}(\sigma^{mn}\varepsilon)_{\alpha\beta}. \tag{2.17}$$

Other (anti)commutation relations follow in a similar way.

Relations (2.16) enable us to define the deformed superspace. It is generated by the usual bosonic and fermionic coordinates (2.1) while the deformation is contained in the new product (2.13). From (2.16) it follows that both fermionic and bosonic part of the superspace are deformed. This is different from [20] where only the fermionic coordinates were deformed.

The deformed infinitesimal SUSY transformation is defined as

$$\delta^\star_\xi F = \big(\xi Q + \bar\xi\bar Q\big)F. \tag{2.18}$$

Since the coproduct (2.9) is undeformed, the usual (undeformed) Leibniz rule follows. Then the $\star$-product of two superfields is again a superfield. Its transformation law is given by

$$\begin{aligned}
\delta^\star_\xi(F\star G) &= \big(\xi Q + \bar\xi\bar Q\big)(F\star G)\\
&= (\delta^\star_\xi F)\star G + F\star(\delta^\star_\xi G).
\end{aligned} \tag{2.19}$$

3 Chiral fields

Since we are interested in possible deformations of the usual[3] Wess-Zumino action, we now analyze chiral fields and their $\star$-products.

A chiral field Φ fulfills $\bar D_{\dot\alpha}\Phi = 0$, where $\bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m_{\ \alpha\dot\alpha}\partial_m$ and $\bar D_{\dot\alpha}$ is related to D_α by the usual complex conjugation. In terms of the component fields the chiral superfield Φ is given by

$$\begin{aligned}
\Phi(x,\theta,\bar\theta) &= A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta H(x) + i\theta\sigma^l\bar\theta(\partial_l A(x))\\
&\quad -\frac{i}{\sqrt2}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\ \alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac14\theta\theta\bar\theta\bar\theta(\Box A(x)).
\end{aligned} \tag{3.1}$$

The $\star$-product of two chiral fields reads

$$\begin{aligned}
\Phi\star\Phi &= \Phi\cdot\Phi - \frac18 C^{\alpha\beta}C^{\gamma\delta}D_\alpha D_\gamma\Phi D_\beta D_\delta\Phi\\
&= \Phi\cdot\Phi - \frac{1}{32}C^2(D^2\Phi)(D^2\Phi)\\
&= A^2 - \frac{C^2}{2}H^2 + 2\sqrt2 A\theta^\alpha\psi_\alpha
\end{aligned}$$

[3]In this paper “usual“ always refers to undeformed, that is to the case $C_{\alpha\beta} = 0$.

$$
\begin{aligned}
&-i\sqrt{2}C^2H\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha)+\theta\theta\Big(2AH-\psi\psi\Big)\\
&+C^2\bar\theta\bar\theta\Big(-H\Box A+\frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)\\
&+i\theta\sigma^m\bar\theta\Big(\partial_m(A^2)+C^2H\partial_mH\Big)\\
&+i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big)\\
&+\frac{\sqrt{2}}{2}\bar\theta\bar\theta C^2(-H\theta\Box\psi+\theta\sigma^m\bar\sigma^n\partial_n\psi\partial_mH)\\
&+\frac{1}{4}\theta\theta\bar\theta\bar\theta\Big(\Box A^2-\frac{1}{2}C^2\Box H^2\Big),
\end{aligned}
\tag{3.2}
$$

where $C^2=C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$. Because of the $\bar\theta$, $\bar\theta\bar\theta$ and the $\theta\bar\theta\bar\theta$ terms (3.2) is not a chiral field. Following the method developed in [20] we decompose the $\star$-products of chiral fields into their irreducible components by using the projectors defined in [21]. The antichiral, chiral and transversal projectors are defined as follows

$$
P_1=\frac{1}{16}\frac{D^2\bar D^2}{\Box}, \tag{3.3}
$$

$$
P_2=\frac{1}{16}\frac{\bar D^2D^2}{\Box}, \tag{3.4}
$$

$$
P_T=-\frac{1}{8}\frac{D\bar D^2D}{\Box}. \tag{3.5}
$$

The chiral part of (3.2) is undeformed and it is given by (for details we refer to [20])

$$
\begin{aligned}
P_2(\Phi\star\Phi)&=\Phi\Phi\\
&=A^2+2\sqrt{2}A\theta^\alpha\psi_\alpha+\theta\theta\Big(2AH-\psi\psi\Big)\\
&\quad+i\theta\sigma^m\bar\theta\Big(\partial_m(A^2)\Big)+i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big)\\
&\quad+\frac{1}{4}\theta\theta\bar\theta\bar\theta\Box A^2.
\end{aligned}
\tag{3.6}
$$

The antichiral part reads

$$
\begin{aligned}
P_1(\Phi\star\Phi)=&-\frac{C^2}{2}H^2-i\sqrt{2}C^2H\bar\theta\bar\sigma^m\partial_m\psi+C^2\bar\theta\bar\theta\Big(-H\Box A+\frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)\\
&+i\theta\sigma^m\bar\theta C^2H\partial_mH+\frac{\sqrt{2}}{2}\bar\theta\bar\theta C^2(-H\theta\Box\psi+\theta\sigma^m\bar\sigma^n\partial_n\psi\partial_mH)\\
&-\frac{1}{8}\theta\theta\bar\theta\bar\theta C^2\Box H^2.
\end{aligned}
\tag{3.7}
$$

In this case there is no transverse part of $\Phi\star\Phi$,

$$
P_T(\Phi\star\Phi)=0. \tag{3.8}
$$

Next, we calculate the $\star$-product of three chiral fields. The following identity applies

$$
\begin{aligned}
(\Phi\star\Phi)\star\Phi&=(\Phi\cdot\Phi+P_1(\Phi\star\Phi))\star\Phi\\
&=\Phi\Phi\Phi-\frac{1}{32}C^2D^2(\Phi\Phi)D^2\Phi+P_1(\Phi\star\Phi)\Phi,
\end{aligned}
\tag{3.9}
$$

with

$$
\begin{aligned}
-\frac{1}{32}C^2D^2(\Phi\Phi)D^2\Phi \,=\, C^2\Big[& -AH^2+\frac{1}{2}H(\psi\psi)-i\sqrt{2}AH\bar\theta\bar\sigma^n(\partial_n\psi) \\
& -i\sqrt{2}(\bar\theta\bar\sigma^m\partial_m(A\psi))H+\frac{i\sqrt{2}}{2}(\bar\theta\bar\sigma^n\partial_n\psi)(\psi\psi) \\
& +\bar\theta\bar\theta\Big(-AH\Box A-\frac{1}{2}H\Box A^2+\frac{1}{2}\psi\psi\Box A \\
& \qquad +\partial_m(A\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big) \\
& +i\theta\sigma^m\bar\theta\partial_m\Big(AH^2-\frac{1}{2}H\psi\psi\Big) \\
& +\frac{\sqrt{2}}{2}(\bar\theta\bar\theta)\Big(-AH\theta\Box\psi+\frac{1}{2}(\theta\Box\psi)(\psi\psi)-H\theta\Box(A\psi) \\
& +\theta\sigma^n\bar\sigma^m\partial_m(A\psi)(\partial_nH)+\frac{1}{2}\theta\sigma^l\bar\sigma^m(\partial_m\psi)\partial_l(2AH-\psi\psi)\Big) \\
& +\frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\Big(-AH^2+\frac{1}{2}(\psi\psi)H\Big)\Big]
\end{aligned} \tag{3.10}
$$

and

$$
\begin{aligned}
P_1(\Phi\star\Phi)\star\Phi \,=\, & P_1(\Phi\star\Phi)\cdot\Phi \\
=\, & C^2\Big[-\frac{1}{2}AH^2-\frac{\sqrt{2}}{2}\theta\psi H^2-\frac{1}{2}\theta\theta H^3-i\sqrt{2}(\bar\theta\bar\sigma^m(\partial_m\psi))AH \\
& +(\bar\theta\bar\theta)\Big(-HA\Box A+\frac{1}{2}A(\partial_l\psi)\sigma^l\bar\sigma^m(\partial_m\psi)\Big) \\
& +i(\theta\sigma^l\bar\theta)\Big(\frac{1}{2}A(\partial_lH^2)-\frac{1}{2}H^2(\partial_lA)+(\psi\sigma_l\bar\sigma^m(\partial_m\psi))H\Big) \\
& +i\sqrt{2}\Big(\frac{1}{4}(\theta\theta)(\bar\theta\bar\sigma^l\psi)(\partial_lH^2)-\frac{5}{4}(\theta\theta)(\bar\theta\bar\sigma^m(\partial_m\psi))H^2\Big) \\
& +\frac{\sqrt{2}}{2}\bar\theta\bar\theta\Big(\theta\sigma^m\bar\sigma^n\partial_m(H\partial_n\psi)A-2(\theta\psi)\Big(H\Box A-\frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big) \\
& \qquad -(H\partial_lA)\theta\sigma^l\bar\sigma^m(\partial_m\psi)\Big) \\
& +\theta\theta\bar\theta\bar\theta\Big(-\frac{1}{8}A\Box H^2-\frac{9}{8}H^2\Box A-\frac{1}{2}\psi\sigma^m\bar\sigma^n\partial_m(H\partial_n\psi) \\
& +H(\partial_m\psi)\sigma^m\bar\sigma^n(\partial_n\psi)+\frac{1}{4}(\partial_mA)(\partial^mH^2)\Big)\Big].
\end{aligned} \tag{3.11}
$$

It is easy to see that

$$
P_2(\Phi\star\Phi)\star\Phi=\Phi\Phi\Phi-\frac{1}{32}C^2D^2(\Phi\Phi)D^2\Phi. \tag{3.12}
$$

The projections are given by

$$
\begin{aligned}
P_1(P_2(\Phi\star\Phi)\star\Phi) \,&=\, -\frac{1}{32}C^2D^2(\Phi\Phi)D^2\Phi \\
P_2(P_2(\Phi\star\Phi)\star\Phi) \,&=\, \Phi\Phi\Phi.
\end{aligned} \tag{3.13}
$$

4 Invariants

Let us now examine the transformation laws under the deformed SUSY transformations (2.18) of terms which could be relevant for the construction of a SUSY invariant action.

There are two quadratic (in the number of fields) invariants,[4] I_1 and I_2:

$$
\begin{aligned}
I_1 &= P_2(\Phi\star\Phi)\Big|_{\theta\theta} = 2AH - \psi\psi, &(4.1)\\
I_2 &= P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta} = -C^2\Big(H\Box A - \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big). &(4.2)
\end{aligned}
$$

Their transformation laws are given by

$$
\begin{aligned}
\delta^\star_\xi I_1 &= 2i\sqrt{2}\bar\xi\bar\sigma^m\partial_m(A\psi), &(4.3)\\
\delta^\star_\xi I_2 &= \sqrt{2}C^2\xi\sigma^m\bar\sigma^n\partial_m(H(\partial_n\psi)). &(4.4)
\end{aligned}
$$

Looking at cubic terms we see that there are more candidates for possible invariants. The first two are I_3 and I_4:

$$
\begin{aligned}
I_3 &= P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} = 3(A^2H - A\psi\psi), &(4.5)\\
I_4 &= P_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta}\\
&= C^2\Big(- AH\Box A - \frac{1}{2}H\Box A^2\\
&\qquad + \frac{1}{2}\psi\psi\Box A + \partial_m(A\psi)\sigma^m\bar\sigma^n(\partial_n\psi)\Big). &(4.6)
\end{aligned}
$$

One can check that they indeed transform as total derivatives. Two more candidates are given by

$$
\begin{aligned}
I_5 &= P_1(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} = C^2\Big(- AH\Box A + \frac{1}{2}A(\partial_l\psi)\sigma^l\bar\sigma^m(\partial_m\psi)\Big), &(4.7)\\
I_6 &= P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} = -\frac{C^2}{2}H^3. &(4.8)
\end{aligned}
$$

However they do not transform as total derivatives

$$
\begin{aligned}
\delta^\star_\xi I_5 &= \frac{C^2}{2}\xi^\alpha\Big(- 2H(A\Box\psi_\alpha + \psi_\alpha\Box A) + 2(\sigma^m\bar\sigma^l)_\alpha{}^\beta(\partial_l\psi_\beta)(\partial_m H)A\\
&\qquad + \psi_\alpha(\partial_l\psi)\sigma^l\bar\sigma^m(\partial_m\psi)\Big) &(4.9)\\
&\neq \partial_m(\dots),\\
\delta^\star_\xi I_6 &= -\frac{3i}{\sqrt{2}}C^2\bar\xi\bar\sigma^m(\partial_m\psi)H^2 &(4.10)
\end{aligned}
$$

[4]Strictly speaking, terms I_1 and I_2 are invariant only under the integral $\int d^4x$, that is when included in an action. Since the construction of an invariant action is our aim, we continue with this abuse of notation and call "invariant" all terms that under SUSY transformations transform as total derivatives.

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$$\neq \partial_m(\dots).$$

Inclusion of these terms will not lead to a SUSY invariant action. The last candidate for a cubic invariant is I_7:

$$I_7 = P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta} = -\frac{C^2}{16}\Big(A\Box H^2 + 5H^2\Box A \\ -4H(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi) + 2\psi\sigma^m\bar\sigma^l\partial_m(H(\partial_l\psi))\Big). \tag{4.11}$$

Since we are interested in equations of motion we omitted a term which is a total derivative in (4.11). Note also that the terms $P_1(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ and $P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ are equal up to a total derivative term and therefore lead to the same equations of motion. Since I_7 is the highest component of a superfield, under (2.18) it transforms as a total derivative and can be included in a SUSY invariant action.

5 SUSY invariant Wess-Zumino model

In order to write the SUSY invariant action we collect all invariant terms and obtain the following Lagrangian

$$\begin{aligned}\mathcal{L} = \Phi^+\star\Phi\Big|_{\theta\theta\bar\theta\bar\theta} &+ \Bigg[\frac{m}{2}\Big(P_2(\Phi\star\Phi)\Big|_{\theta\theta} + aP_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}\Big) \\ &+\frac{\lambda}{3}\Big(P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} + bP_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} \\ &+2c(P_1+P_2)(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}\Big) + \text{ c.c. }\Bigg],\end{aligned} \tag{5.1}$$

with m, λ, a, b and c real constant parameters. Terms $P_1(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ and $P_2(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ are equal up to a total derivative term and are therefore included with the same coefficient. The action in component fields which follows from (5.1) reads

$$\begin{aligned}S = \int \mathrm{d}^4x\,\Bigg\{&A^*\Box A + i\partial_m\bar\psi\bar\sigma^m\psi + H^*H \\ &+m\Big(AH - \frac{1}{2}\psi\psi\Big) + m\Big(A^*H^* - \frac{1}{2}\bar\psi\bar\psi\Big) \\ &+\lambda(A^2H - A\psi\psi) + \lambda\big((A^*)^2H^* - A^*\bar\psi\bar\psi\big) \\ &+\Bigg[C^2\Bigg(ma_1\Big(\frac{1}{2}\psi\Box\psi - H\Box A\Big) + \lambda a_2\Big(-AH\Box A - \frac{1}{2}H(\Box A^2) \\ &\quad +\frac{1}{2}\psi\psi(\Box A) + A\psi\Box\psi\Big) + \lambda a_3\Big(-\frac{3}{2}H^2\Box A + \frac{3}{2}H(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)\Bigg) \\ &\quad + \text{c.c.}\Bigg]\Bigg\}.\end{aligned} \tag{5.2}$$

The coefficients a, b and c are related to a_1, a_2 and a_3: $a/2 = a_1$, $b/3 = a_2$ and $c = a_3$. Note that (5.2) is the full action, i.e. no higher order terms in the deformation parameter $C^{\alpha\beta}$ appear.

Varying the action (5.2) with respect to the fields H and H^* we obtain the equations of motion for these fields

$$\begin{aligned} H^* &= -mA - \lambda A^2 + ma_1 C^2(\Box A) + \lambda a_2 C^2\Big(A\Box A + \frac{1}{2}(\Box A^2)\Big) \\ &\quad -\frac{3}{2}\lambda a_3 C^2\Big(-2H(\Box A) + (\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big), \qquad (5.3) \\ H &= -mA^* - \lambda(A^*)^2 + ma_1\bar C^2(\Box A^*) + \lambda a_2\bar C^2\Big(A^*\Box A^* + \frac{1}{2}(\Box A^{*2})\Big) \\ &\quad -\frac{3}{2}\lambda a_3\bar C^2\Big(-2H^*(\Box A^*) + (\partial_m\bar\psi)\bar\sigma^m\sigma^l(\partial_l\bar\psi)\Big). \qquad (5.4) \end{aligned}$$

Unlike in the undeformed theory, equations (5.3) and (5.4) are nonlinear in H and H^*. Nevertheless they can be solved

$$\begin{aligned} H^* = &\Big(1-9(\lambda a_3)^2C^2\bar C^2(\Box A^*)(\Box A)\Big)^{-1}\Big\{-mA^* - \lambda(A^*)^2 \\ &+ma_1\bar C^2(\Box A^*) + \lambda a_2\bar C^2\Big(A^*\Box A^* + \frac{1}{2}(\Box A^*)^2\Big) \\ &-3\lambda a_3\bar C^2(\Box A^*)\big(mA+\lambda A^2\big) - \frac{3}{2}\lambda a_3\bar C^2(\partial_m\bar\psi)\bar\sigma^m\sigma^l(\partial_l\bar\psi) \\ &+3\lambda a_3\bar C^2(\Box A^*)\Big[ma_1C^2(\Box A) + \lambda a_2 C^2\Big(A\Box A + \frac{1}{2}(\Box A)^2\Big) \\ &\qquad -\frac{3}{2}\lambda a_3 C^2(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big]\Big\}, \qquad (5.5) \end{aligned}$$

and similarly for H. These solutions we can expand up to second order in the deformation parameter and insert in the action (5.2). The action then becomes

$$S = S_0 + S_2, \qquad (5.6)$$

with

$$\begin{aligned} S_0 &= \int d^4x\,\Big\{A^*\Box A + i(\partial_m\bar\psi)\bar\sigma^m\psi - \frac{m}{2}\big(\psi\psi + \bar\psi\bar\psi\big) - \lambda\big(A^*\bar\psi\bar\psi + A\psi\psi\big) \\ &\qquad -m^2A^*A - m\lambda A(A^*)^2 - m\lambda A^*A^2 - \lambda^2A^2(A^*)^2\Big\}, \qquad (5.7) \\ S_2 &= \int d^4x\,\Big\{C^2ma_1\Big(\frac{1}{2}\psi(\Box\psi) + (\Box A)(mA^* + \lambda(A^*)^2)\Big) \\ &\quad +C^2\lambda a_2\Big(\frac{1}{2}\psi\psi(\Box A) + A\psi(\Box\psi) + (mA^*+\lambda(A^*)^2)\Big(A(\Box A) + \frac{1}{2}(\Box A^2)\Big)\Big) \\ &\quad -\frac{3}{2}C^2\lambda a_3(mA^*+\lambda(A^*)^2)\Big((\Box A)(mA^*+\lambda(A^*)^2) + (\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big) \\ &\quad +\bar C^2ma_1\Big(\frac{1}{2}\bar\psi(\Box\bar\psi) + (\Box A^*)(mA+\lambda A^2)\Big) \qquad (5.8) \\ &\quad +\bar C^2\lambda a_2\Big(\frac{1}{2}\bar\psi\bar\psi(\Box A^*) + A^*\bar\psi(\Box\bar\psi) + (mA+\lambda A^2)\Big(A^*(\Box A^*) + \frac{1}{2}(\Box(A^*)^2)\Big)\Big) \\ &\quad -\frac{3}{2}\bar C^2\lambda a_3(mA+\lambda A^2)\Big((\Box A^*)(mA+\lambda A^2) + (\partial_m\bar\psi)\bar\sigma^m\sigma^l(\partial_l\bar\psi)\Big)\Big\}. \end{aligned}$$

6 Renormalizability properties: two-point Green functions

In this section we investigate some renormalizability properties of our model. Using the background field method [25] and the dimensional reduction [26][5] the divergent part of the effective action up to second order in fields is calculated. Note that we work with the action (5.2) and not with (5.6).

To start with, we rewrite the deformed action (5.2) introducing the real fields S, P, E and G as

$$A = \frac{S + iP}{\sqrt{2}}, \quad H = \frac{E + iG}{\sqrt{2}} \tag{6.1}$$

and the Majorana spinor[6] $\psi_M = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$. The deformation parameter $C_{\alpha\beta}$ can be written in the following way

$$C_{\alpha\beta} = K_{ab}(\sigma^{ab}\varepsilon)_{\alpha\beta}, \quad \bar{C}_{\dot{\alpha}\dot{\beta}} = K^*_{ab}(\varepsilon\bar{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}}. \tag{6.2}$$

Since K_{ab} is a self dual tensor we write it as

$$K_{ab} = \kappa_{ab} + \frac{i}{2}\epsilon_{abcd}\kappa^{cd}, \tag{6.3}$$

where κ_{ab} is a real antisymmetric tensor. In this way we obtain

$$C^2 + \bar{C}^2 = 4\kappa_{ab}\kappa^{ab} \tag{6.4}$$

$$C^2 - \bar{C}^2 = 2i\epsilon_{abcd}\kappa^{ab}\kappa^{cd}. \tag{6.5}$$

In order to simplify our calculation we will assume that $C^2 - \bar{C}^2 = 0$. This choice can be obtained by setting $\kappa_{0i} = 0$.

With all this and introducing $g = \frac{\lambda}{\sqrt{2}}$ the action (5.2) becomes[7]

$$S = S_0 + S_2$$

with

$$\begin{aligned} S_0 &= \int \mathrm{d}^4x \left\{ \frac{1}{2}S\Box S + \frac{1}{2}P\Box P - \frac{1}{2}(i\bar{\psi}\gamma^m\partial_m\psi + m\bar{\psi}\psi) + \frac{1}{2}(E^2 + G^2) \right. \\ &\quad + m(SE - PG) - gS\bar{\psi}\psi + gP\bar{\psi}\gamma^5\psi \\ &\quad \left. + g(ES^2 - EP^2 - 2SPG) \right\}, \end{aligned} \tag{6.7}$$

$$S_2 = C^2 \int \mathrm{d}^4x \left\{ ma_1\left(\frac{1}{2}\bar{\psi}\Box\psi - E\Box S + G\Box P\right)\right.$$

[5] The method of dimensional regularization has a draw-back that it might not preserve the supersymmetry. Therefore one uses a modification of it, the so-called dimensional reduction.

[6] The index M on the Majorana spinors will be omitted in the following formulas.

[7] In the notation of [21] the matrix γ_5 and the Lorentz generators Σ^{mn} are defined as

$$\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3, \quad \Sigma^{mn} = \frac{1}{4}[\gamma^m, \gamma^n]. \tag{6.6}$$

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$$
\begin{aligned}
&+ga_2(PG\Box S - SE\Box S + PE\Box P + SG\Box P \\
&-\frac{1}{2}(S^2\Box E - P^2\Box E - 2SP\Box G) + \frac{1}{2}\bar{\psi}\psi\Box S \\
&-\frac{1}{2}\bar{\psi}\gamma^5\psi\Box P + \bar{\psi}\Box\psi S - \bar{\psi}\gamma^5\Box\psi P) \\
&+\frac{3}{2}ga_3(-E^2\Box S + G^2\Box S + 2EG\Box P - E\partial_m\bar{\psi}\partial^m\psi \\
&+ G\partial_m\bar{\psi}\gamma^5\partial^m\psi - 2\bar{\psi}\Sigma^{mn}\partial_n\psi\partial_m E + 2\bar{\psi}\Sigma^{mn}\gamma^5\partial_n\psi\partial_m G)\Big\}.
\end{aligned}
\tag{6.8}
$$

We split the fields into their classical and quantum parts, for example $E \to E + \mathcal{E}$. The action quadratic in quantum fields is

$$
S^{(2)} = \frac{1}{2}\begin{pmatrix}\bar{\Psi} & \mathcal{S} & \mathcal{P} & \mathcal{E} & \mathcal{G}\end{pmatrix} M \begin{pmatrix}\Psi \\ \mathcal{S} \\ \mathcal{P} \\ \mathcal{E} \\ \mathcal{G}\end{pmatrix}, \tag{6.9}
$$

where $\Psi, \bar{\Psi},\ \mathcal{S},\ \mathcal{P},\ \mathcal{E},\ \mathcal{G}$ are quantum fields. The one loop effective action is then

$$
\Gamma = \frac{i}{2}\mathrm{STr}\ln\Big[1 + (\Box - m^2)^{-1}MC\Big], \tag{6.10}
$$

with

$$
C = \begin{pmatrix}
-i\partial\!\!\!/ + m & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -m & 0 \\
0 & 0 & 1 & 0 & m \\
0 & -m & 0 & \Box & 0 \\
0 & 0 & m & 0 & \Box
\end{pmatrix}. \tag{6.11}
$$

The matrix MC can be decomposed into three parts

$$
MC = N + T + V. \tag{6.12}
$$

The zeroth order (in the deformation parameter) term is given by

$$
N = 2g\begin{pmatrix}
(-S+\gamma^5 P)(-i\partial\!\!\!/ + m) & -\psi & \gamma^5\psi & m\psi & m\gamma^5\psi \\
-\bar{\psi}(-i\partial\!\!\!/ + m) & (E - mS) & -(G + mP) & -mE + S\Box & -mG - P\Box \\
\bar{\psi}\gamma^5(-i\partial\!\!\!/ + m) & -G + mP & -E - mS & mG - P\Box & -mE - S\Box \\
0 & S & -P & -mS & -mP \\
0 & -P & -S & mP & -mS
\end{pmatrix}. \tag{6.13}
$$

The second order term (in the deformation parameter) which contains no fields is

$$
T = ma_1C^2\begin{pmatrix}
\Box(-i\partial\!\!\!/ + m) & 0 & 0 & 0 & 0 \\
0 & m\overleftarrow{\Box} & 0 & -\overleftarrow{\Box}\overrightarrow{\Box} & 0 \\
0 & 0 & m\overleftarrow{\Box} & 0 & \overleftarrow{\Box}\overrightarrow{\Box} \\
0 & -\Box & 0 & m\Box & 0 \\
0 & 0 & \Box & 0 & m\Box
\end{pmatrix}. \tag{6.14}
$$

The matrix V is second order in the deformation parameter and contains classical fields linearly. Its matrix elements are given in appendix A.

The one-loop divergent part of the effective action we calculate up to second order in g, second order in fields (two-point functions) and up to second order in the deformation parameter $C_{\alpha\beta}$. Therefore the effective action is given by

$$\begin{aligned}\Gamma &= \frac{i}{2}\mathrm{STr}\ln\Big[1+(\Box-m^2)^{-1}(N+T+V)\Big]\\ &= \frac{i}{2}\Big[\mathrm{STr}((\Box-m^2)^{-1}(N+T+V))\\ &\quad -\frac{1}{2}\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}N)\\ &\quad -\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}(T+V))\\ &\quad +\mathrm{STr}(((\Box-m^2)^{-1}N)^2(\Box-m^2)^{-1}T)\Big].\end{aligned} \tag{6.15}$$

The calculation of divergent parts of supertraces is tedious but straightforward and here we give only the results. The details are given in appendix B. Denoting $K=\Box-m^2$, we have

$$\mathrm{STr}(K^{-1}(N+T+V)) = 0, \tag{6.16}$$

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}N) &= \frac{ig^2}{\pi^2\epsilon}\int \mathrm{d}^4x\\ &\quad\times\Big[S\Box S+P\Box P-\bar\psi i\partial\!\!\!/\psi+E^2+G^2\Big],\end{aligned} \tag{6.17}$$

$$\mathrm{STr}\Big[K^{-1}NK^{-1}T\Big] = 0, \tag{6.18}$$

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}V) &= -g^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3a_3m^2(-2P\Box G\\ &\quad-\bar\psi\Box\psi+2S\Box E)\\ &\quad+a_2((\Box S)^2+(\Box P)^2+4m^2S\Box S\\ &\quad-\bar\psi i\partial\!\!\!/\Box\psi-2m^2\bar\psi i\partial\!\!\!/\psi+4m^2P\Box P\\ &\quad+4m^2E^2+E\Box E+G\Box G+4m^2G^2)\Big],\end{aligned} \tag{6.19}$$

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}NK^{-1}T) &= \frac{2iC^2a_1m^2g^2}{\pi^2\epsilon}\int \mathrm{d}^4x\\ &\quad\times\Big[S\Box S+P\Box P-\bar\psi i\partial\!\!\!/\psi+E^2+G^2\Big].\end{aligned} \tag{6.20}$$

In (6.19) terms $(\Box S)^2$ and $(\Box P)^2$ appear. Since these terms do not have classical counterparts we take $a_2=0$. Then the divergent part of the one loop effective action (6.15) is given by

$$\begin{aligned}\Gamma_1 = \frac{g^2}{\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[&\frac{1}{4}(S\Box S+P\Box P+\bar\psi i\partial\!\!\!/\psi+E^2+G^2)\\ &+\frac{3}{4}a_3C^2m^2(2P\Box G+\bar\psi\Box\psi-2S\Box E)\\ &-C^2a_1m^2(S\Box S+P\Box P-\bar\psi i\partial\!\!\!/\psi+E^2+G^2)\Big].\end{aligned} \tag{6.21}$$

Let us now discuss the one-loop renormalizability properties of our model. To cancel the divergences we have to add to the classical Lagrangian the counterterms

$$\mathcal{L}_B = \mathcal{L}_0 + \mathcal{L}_2 - \Gamma_1. \tag{6.22}$$

In this way we obtain the bare Lagrangian $\mathcal{L}_B$. It is important to note that the term I_7 in the classical action (5.2) produces divergences proportional to I_2 (compare (6.19) and (6.8)), so both of them are necessary in order to absorb the divergences in the effective action. From the form of the bare Lagrangian we see that all fields are renormalized in the same way:

$$S_0 = \sqrt{Z}S,\ P_0 = \sqrt{Z}P,\ \psi_0 = \sqrt{Z}\psi,\ E_0 = \sqrt{Z}E,\ G_0 = \sqrt{Z}G, \tag{6.23}$$

with

$$Z = 1 - \frac{g^2}{2\pi^2\epsilon}(1 - 4a_1 m^2 C^2). \tag{6.24}$$

The tadpole contributions add up to zero as in the commutative case. Also, $\delta m = 0$, i.e. there are no $\delta m\bar{\psi}\psi$ and $\delta m(SE + PG)$ counterterms. It is obvious that the deformation parameter has to be renormalized too,

$$C_0^2 = \left(1 - \frac{3a_3 g^2}{2\pi^2 a_1\epsilon}\right)C^2\ . \tag{6.25}$$

The present analysis is not complete and we plan to consider the vertex corrections in a forthcoming publication. From the vertex corrections we should draw conclusions about the renormalization of the coupling constant g and about the renormalizability of the full model.

Finally, let us make a comment concerning the non-renormalization theorem. From (6.21) we see that the divergent part of the effective action consists of the usual term $(\Phi^+\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}$ and a new (compared to the undeformed case) term $P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}$. Both terms are expressible as integrals over the whole superspace. In particular, for the new term we have

$$\begin{aligned}
P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta} &= \int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \theta\theta P_1(\Phi\star\Phi) \\
&= -\frac{1}{32}C^2\int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \theta\theta(D^2\Phi)(D^2\Phi) \\
&= \frac{1}{8}C^2\int \mathrm{d}^4x\ \mathrm{d}^2\ \bar\theta \mathrm{d}^2\ \theta\ \Phi(D^2\Phi).
\end{aligned} \tag{6.26}$$

We see that at the level of two-point Green functions there is no need to deform the nonrenormalization theorem. This conclusion is different from [27].

7 Conclusions

In order to see how a deformation by twist of the usual Wess-Zumino model affects its renormalizability properties, we considered a special example of the twist (2.6). Compared with the undeformed SUSY Hopf algebra, the twisted SUSY Hopf algebra is unchanged.

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In particular, the twisted coproduct is undeformed, which leads to the undeformed Leibniz rule (2.19). However, the notion of chirality is lost and we have to apply the method of projectors introduced in [20]. By including all constructed invariants, we formulate a deformation of the usual Wess-Zumino action (5.2). Finally, we discuss some preliminary renormalizability properties of the model. As expected, there are no tadpole diagrams and no mass renormalization counterterms. All fields are renormalized in the same way, which is another property of SUSY invariant theories. As the renormalization of the coupling constant g is concerned, at present we cannot say if it is renormalized and how. However, we see that the freedom in choosing terms in the action is partially fixed by demanding the cancellation of divergences. That request leads to $a_2 = 0$ and additionally we see that both a_1 and a_3 terms are necessary.

Let us remark that the twist (2.6) leads to the $\star$-product (2.13) which has already been discussed in [14]. In that paper the deformed Wess-Zumino Lagrangian has been constructed in two different ways. The difference was present in the interaction terms. Namely, one can take the term $\Phi_\star^3\big|_{\theta\theta}$ which (since $\Phi_\star^3$ is not chiral) breaks 1/2 SUSY; this term is equal to our I_6 (4.8) and was not included in our deformed model (5.2) since it is not SUSY invariant. Adding its complex conjugate breaks the full supersymmetry. The other possibility which was considered in [14] was to take the term $\Phi_\star^3\big|_{\theta\theta\bar\theta\bar\theta}$ as an interaction term. Is is equal to our I_7 (4.11). Since it is the highest component of the superfield $\Phi_\star^3$, it transforms as a total derivative and the action is invariant under the full supersymmetry. However, its commutative limit is zero and it is not a deformation of the usual interaction term. The commutative limit is obtained in [14] by adding the term $(\Phi^+)_\star^3$ which is undeformed and its complex conjugate reproduces the proper commutative limit. We have seen that the action with only the I_7 term is not renormalizable.

Renormalizability of the deformed Wess-Zumino models with the term $\Phi_\star^3 \propto H^3$ was studied, see for example [16]. To make these models renormalizable one has to add additional terms to the original action. The main advantage of our model is the absence of this problem. By including all possible invariants from the beginning we see that no new terms are needed to cancel the divergences that appear. However, our results are not complete since we calculated here only the divergences in the two-point functions. In the forthcoming paper we will consider the vertex contributions and then we will be able to tell if our present conclusions still hold.

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A Matrix elements of V

The matrix elements of V are given by

$$\begin{aligned} V_{11} = gC^2\Big(&a_2(\Box S + 2S\Box - \gamma^5\Box P - 2P\gamma^5\Box) \\ &+3a_3(E\Box + (\partial_m E)\partial^m - (\partial_m G)\gamma^5\partial^m - G\gamma^5\Box \end{aligned}$$

$$
\begin{aligned}
& -2\Sigma^{mn}\partial_m E\partial_n + 2\Sigma^{mn}\gamma^5\partial_m G\partial_n)\Big)(-i\partial\!\!\!/ + m), && \text{(A.1)}\\
V_{12} &= gC^2\Big[a_2\Big(\psi\Box + 2\Box\psi\Big) - 3a_3 m\Big(\Box\psi + \partial_m\psi\partial^m - 2\Sigma^{mn}\partial_n\psi\partial_m\Big)\Big], && \text{(A.2)}\\
V_{13} &= gC^2\Big[a_2\Big(-\gamma^5\psi\Box - 2\gamma^5\Box\psi\Big)\\
&\quad +3a_3 m\Big(-\gamma^5\Box\psi - \gamma^5\partial_m\psi\partial^m + 2\Sigma^{mn}\gamma^5\partial_n\psi\partial_m\Big)\Big], && \text{(A.3)}\\
V_{14} &= gC^2\Big[-ma_2\Big(\psi\Box + 2\Box\psi\Big) + 3a_3\Big(\Box\psi\\
&\quad +\partial_m\psi\partial^m - 2\Sigma^{mn}\partial_n\psi\partial_m\Big)\Box\Big], && \text{(A.4)}\\
V_{15} &= gC^2\Big[ma_2\Big(-\gamma^5\psi\Box - 2\gamma^5\Box\psi\Big)\\
&\quad +3a_3 m\Big(-\gamma^5\Box\psi - \gamma^5\partial_m\psi\partial^m + 2\Sigma^{mn}\gamma^5\partial_n\psi\partial_m\Big)\Box\Big], && \text{(A.5)}\\
V_{21} &= gC^2\Big[a_2\Big(\overleftarrow{\Box}\bar\psi + 2\bar\psi\Box\Big)(-i\partial\!\!\!/ + m), && \text{(A.6)}\\
V_{22} &= gC^2\Big[-a_2(2E\Box + \Box E) + ma_2(\Box S + S\Box + \overleftarrow{\Box}S) + 3ma_3\overleftarrow{\Box}E\Big], && \text{(A.7)}\\
V_{23} &= gC^2\Big[a_2(\Box G + G\Box + \overleftarrow{\Box}G)\\
&\quad +ma_2(\Box P + P\Box + \overleftarrow{\Box}P) + 3ma_3\overleftarrow{\Box}G\Big], && \text{(A.8)}\\
V_{24} &= gC^2\Big[ma_2(2E\Box + \Box E) - a_2(\Box S + S\Box + \overleftarrow{\Box}S)\Box - 3a_3\overleftarrow{\Box}E\Box\Big], && \text{(A.9)}\\
V_{25} &= gC^2\Big[ma_2(\Box G + G\Box + \overleftarrow{\Box}G)\\
&\quad +a_2(\Box P + P\Box + \overleftarrow{\Box}P)\Box + 3a_3\overleftarrow{\Box}G\Box\Big], && \text{(A.10)}\\
V_{31} &= gC^2\Big[a_2\Big(-\overleftarrow{\Box}\bar\psi\gamma^5 - 2\bar\psi\gamma^5\Box\Big)(-i\partial\!\!\!/ + m)\Big], && \text{(A.11)}\\
V_{32} &= gC^2\Big[a_2(\Box G + G\Box + \overleftarrow{\Box}G) - a_2(\Box P + P\Box + \overleftarrow{\Box}P) - 3ma_3\overleftarrow{\Box}G\Big], && \text{(A.12)}\\
V_{33} &= gC^2\Big[a_2(2E\Box + \Box E) + ma_2(\Box S + S\Box + \overleftarrow{\Box}S) + 3ma_3\overleftarrow{\Box}E\Big], && \text{(A.13)}\\
V_{34} &= gC^2\Big[-a_2 m(\Box G + G\Box + \overleftarrow{\Box}G)\\
&\quad +a_2(\Box P + P\Box + \overleftarrow{\Box}P)\Box + 3a_3\overleftarrow{\Box}G\Box\Big], && \text{(A.14)}\\
V_{35} &= gC^2\Big[ma_2(2E\Box + \Box E) + a_2(\Box S + S\Box + \overleftarrow{\Box}S)\Box + 3a_3\overleftarrow{\Box}E\Box\Big], && \text{(A.15)}\\
V_{41} &= gC^2\Big[3a_3\Big(-\partial^m\bar\psi\partial_m - 2\partial_m\bar\psi\Sigma^{mn}\partial_n\Big)(-i\partial\!\!\!/ + m)\Big], && \text{(A.16)}\\
V_{42} &= gC^2\Big[-a_2(\Box S + S\Box + \overleftarrow{\Box}S) + 3a_3(m\Box S - E\Box)\Big], && \text{(A.17)}\\
V_{43} &= gC^2\Big[a_2(\Box P + P\Box + \overleftarrow{\Box}P) + 3a_3(m\Box P + G\Box)\Big], && \text{(A.18)}\\
V_{44} &= gC^2\Big[a_2 gm(\Box S + S\Box + \overleftarrow{\Box}S) + 3a_3(-\Box S\Box + mE\Box)\Big], && \text{(A.19)}\\
V_{45} &= gC^2\Big[ma_2(\Box P + P\Box + \overleftarrow{\Box}P) + 3a_3((\Box P)\Box + mG\Box)\Big], && \text{(A.20)}\\
V_{51} &= gC^2\Big[3a_3\Big(\partial^m\bar\psi\gamma^5\partial_m - 2\partial_m\bar\psi\Sigma^{mn}\partial_n\Big)(-i\partial\!\!\!/ + m)\Big], && \text{(A.21)}
\end{aligned}
$$

$$V_{52} = gC^2\Big[a_2(\Box P + P\Box + \overleftarrow{\Box} P) + 3a_3(-m\Box P + G\Box)\Big], \tag{A.22}$$

$$V_{53} = gC^2\Big[a_2(\Box S + S\Box + \overleftarrow{\Box} S) + 3ma_3(m\Box S + E\Box)\Big], \tag{A.23}$$

$$V_{54} = -gC^2\Big[ma_2(\Box P + P\Box + \overleftarrow{\Box} P) - 3a_3(-mG\Box + (\Box P)\Box)\Big], \tag{A.24}$$

$$V_{55} = gC^2\Big[ma_2(\Box S + S\Box + \overleftarrow{\Box} S) + 3a_3(mE\Box + (\Box S))\Box\Big]. \tag{A.25}$$

B Calculation of supertraces

Here we calculate the divergent parts of two supertraces: $\mathrm{STr}(K^{-1}NK^{-1}V)$ and $\mathrm{STr}(K^{-1}NK^{-1}NK^{-1}T)$. The following general formulas for the divergent parts of traces are used

$$\mathrm{Tr}(K^{-1}fK^{-1}g) = \frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, fg, \tag{B.1}$$

$$\mathrm{Tr}(\partial_n K^{-1}fK^{-1}g) = \frac{i}{16\pi^2\epsilon}\int \mathrm{d}^4x\, \partial_n fg, \tag{B.2}$$

$$\begin{aligned}\mathrm{Tr}(\partial_n K^{-1}f\partial_m K^{-1}g) &= -\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \\ &\quad\times\left(\frac{1}{6}\partial_m\partial_n fg + \frac{1}{12}\eta_{mn}\Box fg - \frac{1}{2}\eta_{mn}m^2 fg\right),\end{aligned} \tag{B.3}$$

$$\mathrm{Tr}(K^{-1}f\partial_a K^{-1}g\partial_b K^{-1}h) = \frac{i}{32\pi^2\epsilon}\eta_{ab}\int \mathrm{d}^4x\, fgh, \tag{B.4}$$

$$\mathrm{Tr}(K^{-1}f) = \frac{i}{8\pi^2\epsilon}m^2\int \mathrm{d}^4x\, f, \tag{B.5}$$

$$\mathrm{Tr}(\partial_a K^{-1}f) = 0, \tag{B.6}$$

$$\mathrm{Tr}(\Box K^{-1}f) = \frac{im^4}{8\pi^2\epsilon}\int \mathrm{d}^4x\, f, \tag{B.7}$$

$$\mathrm{Tr}(\Box^2 K^{-1}f) = \frac{im^6}{16\pi^2\epsilon}\int \mathrm{d}^4x\, f. \tag{B.8}$$

- $\mathrm{STr}(K^{-1}NK^{-1}V)$

  Using the definition of Supertrace we obtain

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}V) = &-\sum_i \mathrm{Tr}(K^{-1}N_{1i}K^{-1}V_{i1}) \\ &+\sum_i \mathrm{Tr}(K^{-1}N_{2i}K^{-1}V_{i2}) + \ldots \\ &+\sum_i \mathrm{Tr}(K^{-1}N_{5i}K^{-1}V_{i5}).\end{aligned} \tag{B.9}$$

  The terms in (B.9) are

$$\mathrm{Tr}\Big[K^{-1}N_{11}K^{-1}V_{11}\Big]$$

$$= g^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x \Big[a_2((\Box S)^2 + (\Box P)^2 - 4m^2 S\Box S - 20m^4S^2 - 4m^4P^2)$$
$$+3a_3(-m^2P\Box G - 10m^4SE + m^2S\Box E - 2m^4PG)\Big], \tag{B.10}$$

$$\mathrm{Tr}\Big[K^{-1}N_{23}K^{-1}V_{32} + K^{-1}N_{32}K^{-1}V_{23}\Big]$$
$$= \frac{ig^2C^2}{4\pi^2\epsilon}\int \mathrm{d}^4x \Big[-2a_2(G\Box G + 4m^2G^2)$$
$$+2m^2a_2(P\Box P + 4m^2P^2) + 12a_3m^4PG\Big], \tag{B.11}$$

$$\mathrm{Tr}\Big[K^{-1}N_{45}K^{-1}V_{54} + K^{-1}N_{54}K^{-1}V_{45}\Big]$$
$$= \frac{i}{2\pi^2\epsilon}mg^2C^2\int \mathrm{d}^4x \Big[a_2m(P\Box P + 4m^2P^2) + 6a_3m^3PG\Big], \tag{B.12}$$

$$\mathrm{Tr}\Big[K^{-1}V_{41}K^{-1}N_{14} + K^{-1}V_{51}K^{-1}N_{15}\Big]$$
$$= 3g^2C^2a_3m^2\frac{i}{4\pi^2\epsilon}\int \mathrm{d}^4x\, \bar\psi\Box\psi, \tag{B.13}$$

$$\mathrm{Tr}\Big[K^{-1}N_{22}K^{-1}V_{22} + K^{-1}N_{33}K^{-1}V_{33}\Big]$$
$$= g^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x \Big[-2a_2\Big(2m^2E^2 + \frac{1}{2}E\Box E\Big)$$
$$-m^2a_2(S\Box S + 4m^2S^2) - 6a_3m^4SE\Big], \tag{B.14}$$

$$\mathrm{Tr}\Big[K^{-1}N_{44}K^{-1}V_{44} + K^{-1}N_{55}K^{-1}V_{55}\Big]$$
$$= -g^2m^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x \Big[a_2(4m^2S^2 + S\Box S) + 6a_3m^2SE\Big]. \tag{B.15}$$

$$\mathrm{Tr}\Big[K^{-1}N_{42}K^{-1}V_{24} + K^{-1}N_{24}K^{-1}V_{42} + K^{-1}N_{35}K^{-1}V_{53} + K^{-1}N_{53}K^{-1}V_{35}\Big]$$
$$= \frac{3ig^2C^2}{2\pi^2\epsilon}\int \mathrm{d}^4x \Big[-2a_2(m^2S\Box S + 2m^4S^2)$$
$$-m^2a_3(6m^2SE + E\Box S)\Big], \tag{B.16}$$

$$\mathrm{Tr}\Big[K^{-1}N_{25}K^{-1}V_{52} + K^{-1}N_{52}K^{-1}V_{25} + K^{-1}N_{34}K^{-1}V_{43} + K^{-1}N_{43}K^{-1}V_{34}\Big]$$
$$= \frac{3i}{2\pi^2\epsilon}g^2C^2\int \mathrm{d}^4x \Big[-2a_2(m^2P\Box P + 2m^4P^2)$$
$$-m^2a_3(6m^2PG - G\Box P)\Big], \tag{B.17}$$

$$\mathrm{Tr}\Big[K^{-1}N_{21}K^{-1}V_{12} - K^{-1}N_{12}K^{-1}V_{21} + K^{-1}N_{31}K^{-1}V_{13} - K^{-1}N_{13}K^{-1}V_{31}\Big]$$
$$= g^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x \Big[a_2(2im^2\bar\psi\partial\!\!\!/\psi + i\bar\psi\partial\!\!\!/\Box\psi)$$
$$+\frac{3}{2}a_3m^2\bar\psi\Box\psi\Big]. \tag{B.18}$$

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Adding all the terms (B.10)–(B.15) we obtain

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}V) = & -g^2C^2\frac{i}{2\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3a_3m^2(-2P\Box G\\ & -\bar\psi\Box\psi + 2S\Box E)\\ & +a_2((\Box S)^2+(\Box P)^2+4m^2S\Box S\\ & -\bar\psi i\partial\!\!\!/\Box\psi - 2m^2\bar\psi i\partial\!\!\!/\psi + 4m^2P\Box P\\ & +4m^2E^2+E\Box E+G\Box G+4m^2G^2)\Big].\end{aligned} \tag{B.19}$$

- $\mathrm{STr}(K^{-1}NK^{-1}NK^{-1}T)$

Again, from the definition of Supertrace it follows

$$\begin{aligned}\mathrm{STr}(K^{-1}NK^{-1}NK^{-1}T) = & -\mathrm{Tr}(K^{-1}N_{1i}K^{-1}N_{ij}K^{-1}T_{j1})\\ & +\mathrm{Tr}(K^{-1}N_{2i}K^{-1}N_{ij}K^{-1}T_{j2})\\ & +\dots\\ & +\mathrm{Tr}(K^{-1}N_{5i}K^{-1}N_{ij}K^{-1}T_{j5}).\end{aligned} \tag{B.20}$$

The divergences appearing in (B.20) are

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{11}K^{-1}N_{11}\Box(-i\partial\!\!\!/+m))\\ &\quad = 4g^2m\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[-4S\Box S\\ &\qquad -4P\Box P+40m^2S^2+8m^2P^2\Big],\end{aligned} \tag{B.21}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{22}K^{-1}N_{24}K^{-1}\Box)\\ &\quad = \frac{4g^2i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[-mE^2+4m^2SE-3m^3S^2\Big],\end{aligned} \tag{B.22}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{23}K^{-1}N_{34}K^{-1}\Box)\\ &\quad = \frac{4g^2i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[-mG^2+2mGP+3m^2P^2\Big],\end{aligned} \tag{B.23}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{24}K^{-1}N_{44}K^{-1}\Box)\\ &\quad = \frac{4g^2i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[m^2SE-3m^3S^2\Big],\end{aligned} \tag{B.24}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{25}K^{-1}N_{54}K^{-1}\Box)\\ &\quad = -4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3m^3P^2+m^2PG\Big],\end{aligned} \tag{B.25}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{32}K^{-1}N_{25}K^{-1}\Box)\\ &\quad = \frac{4g^2i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[2m^2PG+mG^2-3m^3P^2\Big],\end{aligned} \tag{B.26}$$

$$\begin{aligned}&\mathrm{Tr}(K^{-1}N_{33}K^{-1}N_{35}K^{-1}\Box)\\ &\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[4m^2ES+3m^3S^2+mE^2\Big],\end{aligned} \tag{B.27}$$

$$\begin{aligned}
&\mathrm{Tr}(K^{-1}N_{34}K^{-1}N_{45}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3m^3P^2 - m^2GP\Big], && \text{(B.28)}\\
&\mathrm{Tr}(K^{-1}N_{35}K^{-1}N_{55}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[m^2ES + 3m^3S^2\Big], && \text{(B.29)}\\
&m\mathrm{Tr}(K^{-1}N_{24}K^{-1}N_{42}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3m^3S^2 - m^2ES\Big], && \text{(B.30)}\\
&m\mathrm{Tr}(K^{-1}N_{25}K^{-1}N_{52}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[3m^3P^2 + m^2PG\Big], && \text{(B.31)}\\
&m\mathrm{Tr}(K^{-1}N_{35}K^{-1}N_{53}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[m^2ES + 3m^3S^2\Big], && \text{(B.32)}\\
&m\mathrm{Tr}(K^{-1}N_{34}K^{-1}N_{43}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[-m^2GP + 3m^3P^2\Big], && \text{(B.33)}\\
&\mathrm{Tr}(K^{-1}N_{33}K^{-1}N_{33}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,(E + mS)^2, && \text{(B.34)}\\
&\mathrm{Tr}(K^{-1}N_{32}K^{-1}N_{23}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[mG^2 - m^3P^2\Big], && \text{(B.35)}\\
&m\mathrm{Tr}(K^{-1}N_{35}K^{-1}N_{53}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[m^2ES + 3m^3S^2\Big], && \text{(B.36)}\\
&\mathrm{Tr}(K^{-1}N_{54}K^{-1}N_{45}K^{-1}\Box)\\
&\quad = -4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,m^3P^2, && \text{(B.37)}\\
&\mathrm{Tr}(K^{-1}N_{55}K^{-1}N_{55}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,m^3S^2, && \text{(B.38)}\\
&\mathrm{Tr}(K^{-1}N_{22}K^{-1}N_{22}K^{-1}\Box)\\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\,(E - mS)^2, && \text{(B.39)}\\
&\mathrm{Tr}(K^{-1}N_{21}K^{-1}N_{14}K^{-1}\Box)\\
&\quad = 4g^2\frac{im}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[\frac{i}{2}\bar\psi\partial\!\!\!/\psi - m\bar\psi\psi\Big], && \text{(B.40)}\\
&\mathrm{Tr}(K^{-1}N_{31}K^{-1}N_{15}K^{-1}\Box)\\
&\quad = -4g^2\frac{im}{8\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[\frac{i}{2}\bar\psi\partial\!\!\!/\psi + m\bar\psi\psi\Big], && \text{(B.41)}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Tr}(K^{-1}N_{31}K^{-1}N_{13}K^{-1}\Box) \\
&\quad = -4mg^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[\frac{i}{2}\bar\psi\partial\!\!\!/\psi + m\bar\psi\psi\right], && \text{(B.42)}\\
&\mathrm{Tr}(K^{-1}N_{21}K^{-1}N_{12}K^{-1}\Box) \\
&\quad = 4g^2m\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[-\frac{i}{2}\bar\psi\partial\!\!\!/\psi + m\bar\psi\psi\right], && \text{(B.43)}\\
&\mathrm{Tr}(K^{-1}N_{13}K^{-1}N_{31}K^{-1}(-i\partial\!\!\!/ + m)\Box) + \mathrm{Tr}(K^{-1}N_{12}K^{-1}N_{21}K^{-1}(-i\partial\!\!\!/ + m)\Box) \\
&\quad = 8g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, i\bar\psi\partial\!\!\!/\psi, && \text{(B.44)}\\
&\mathrm{Tr}(K^{-1}N_{42}K^{-1}N_{22}\Box K^{-1}\Box) \\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[3m^2SE - 3m^3S^2\right], && \text{(B.45)}\\
&\mathrm{Tr}(K^{-1}N_{43}K^{-1}N_{32}\Box K^{-1}\Box) \\
&\quad = 4g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[3m^2PG - 3m^3P^2\right], && \text{(B.46)}\\
&\mathrm{Tr}(K^{-1}N_{44}K^{-1}N_{42}\Box K^{-1}\Box) \\
&\quad = -12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, m^3S^2, && \text{(B.47)}\\
&\mathrm{Tr}(K^{-1}N_{45}K^{-1}N_{52}\Box K^{-1}\Box) \\
&\quad = 12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, m^3P^2, && \text{(B.48)}\\
&\mathrm{Tr}(K^{-1}N_{54}K^{-1}N_{43}\Box K^{-1}\Box) \\
&\quad = -12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, m^3P^2, && \text{(B.49)}\\
&\mathrm{Tr}(K^{-1}N_{55}K^{-1}N_{53}\Box K^{-1}\Box) \\
&\quad = 12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x\, m^3S^2, && \text{(B.50)}\\
&\mathrm{Tr}(K^{-1}N_{52}K^{-1}N_{23}\Box K^{-1}\Box) \\
&\quad = 12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[m^2PG + m^3P^2\right], && \text{(B.51)}\\
&\mathrm{Tr}(K^{-1}N_{53}K^{-1}N_{33}\Box K^{-1}\Box) \\
&\quad = 12g^2\frac{i}{8\pi^2\epsilon}\int \mathrm{d}^4x \left[m^2SE + m^3S^2\right]. && \text{(B.52)}
\end{aligned}$$

Summing the terms (B.21)–(B.52) we obtain

$$\begin{aligned}
\mathrm{STr}(K^{-1}NK^{-1}NK^{-1}T) = {}& \frac{2ia_1C^2m^2g^2}{\pi^2\epsilon} && \text{(B.53)}\\
& \times \int \mathrm{d}^4x \left[S\Box S + P\Box P - \bar\psi i\partial\!\!\!/\psi + E^2 + G^2\right].
\end{aligned}$$

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